

# Multi-arm Group Sequential Designs with a Simultaneous Stopping Rule

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ASTERIX Project - <http://www.asterix-fp7.eu/>

# Objectives of multi-arm multi-stage trials

**Aim:** Comparison of several treatments to a common control

**Compared to separate, fixed sample two-armed trials**

- less patients needed
- larger number of patients is randomised to experimental treatments
- possibility to stop early for efficacy or futility

**Objective:** Identify **all** treatments that are superior to control

**Objective:** Identify **at least one** treatment that is superior to control

**Which stopping rule?**

## Design setup: group sequential Dunnett test

- Comparison of two treatments to a control
- Normal endpoints, variance known
- One sided tests:  $H_A : \mu_A - \mu_C \leq 0$  and  $H_B : \mu_B - \mu_C \leq 0$
- Control of the FamilyWise Error Rate (FWER) = 0.025
- Two stage group sequential trial: one interim analysis at  $\frac{N_{max}}{2}$
- $Z_{A,i}$ ,  $Z_{B,i}$  are the cumulative z-statistics at stage  $i=1,2$

## Classical group sequential Dunnett tests with “separate stopping”

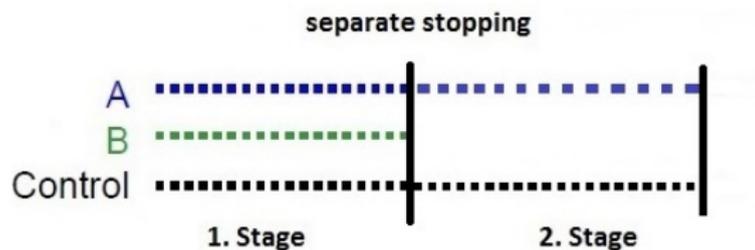
## Classical group sequential Dunnett tests

**Objective:** Identify all treatments that are superior to control

**"separate stopping rule":**

Treatment arms, for which a stopping boundary is crossed, stop.

E.g.:



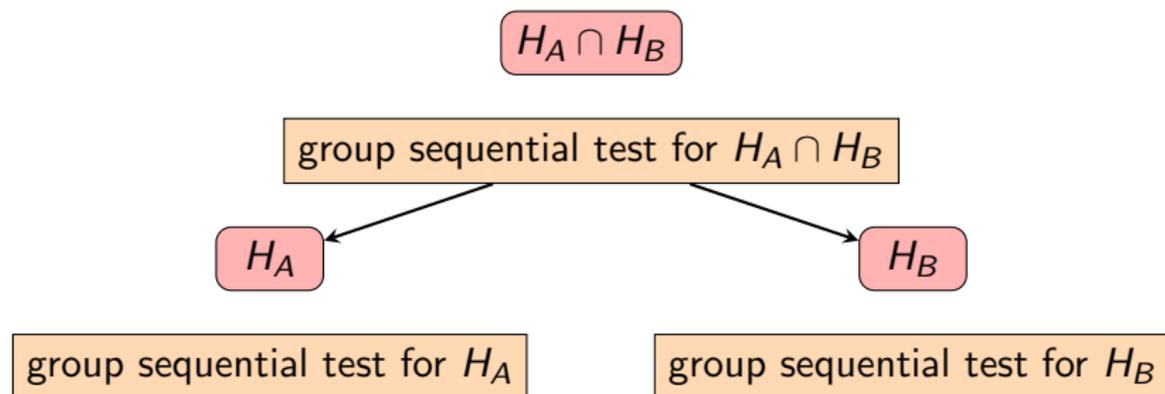
→  $H_B$  is rejected at interim

→ A can go on and is tested again at the end

Magirr, Jaki, Whitehead (2012)

## Control of the FWER: Closed group sequential tests

**Local group sequential tests for  $H_A \cap H_B$  and  $H_A, H_B$  are needed!**



A hypothesis is rejected at FWER  $\alpha$  if the intersection hypothesis and the corresponding elementary hypothesis are rejected locally at level  $\alpha$ .

## Control of the FWER: Closed group sequential tests

$$H_A \cap H_B$$

Reject if  $\max(Z_{A,1}, Z_{B,1}) > u_1$  or  $\max(Z_{A,2}, Z_{B,2}) > u_2$

 $H_A$ 
 $H_B$ 

Reject if  $Z_{A,1} > v_1$  or  $Z_{A,2} > v_2$

Reject if  $Z_{B,1} > v_1$  or  $Z_{B,2} > v_2$

$u_1, u_2$ ...global boundaries

$v_1, v_2$ ...elementary boundaries

Koenig, Brannath, Bretz and Posch (2008)

Xi, Tamhane (2015)

Maurer, Bretz (2013)

## Group sequential Dunnett tests with “simultaneous stopping”

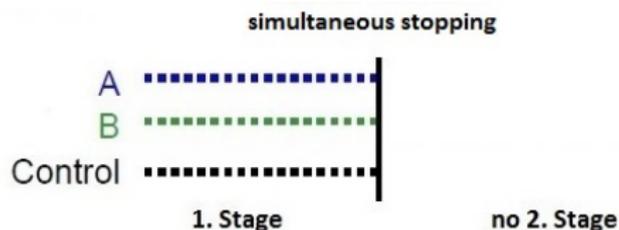
## Group sequential simultaneous stopping designs

**"simultaneous stopping rule":**

If at least one rejection boundary is crossed, the whole trial stops.

**Objective:** Identify at least one treatment that is superior to control

If, e.g.,  $H_B$  is rejected at interim then the trial is stopped:



# Simultaneous versus Separate Stopping

- The **FWER** is controlled when using the boundaries of the separate stopping design.
- The **expected sample size (ESS)** is lower compared to separate stopping designs.
- The **power to reject**
  - **any** null hypothesis is the **same** as for separate stopping designs.
  - **both** null hypotheses is **lower** than for separate stopping designs.

→ **Trade-off between ESS and conjunctive power**

# Construction of efficient simultaneous stopping designs

- 1 Can one **relax the boundaries** when stopping simultaneously?
- 2 How large is the impact on **ESS and power** when stopping simultaneously or separately?
- 3 How to **optimize** the critical boundaries for either stopping rule?

## Question 1: Relaxation of boundaries?

### For simultaneous stopping:

- For simultaneous stopping there is no second stage test if one of the null hypotheses can already be rejected at interim.
- The boundaries  $u_1, u_2$  for the local test of  $H_A \cap H_B$  cannot be relaxed.
- The boundaries  $v_1, v_2$  for the local test of  $H_j$  can be relaxed.

### Intuitive explanation

If, e.g.,  $H_B$  is rejected at interim, but  $H_A$  not,  $H_A$  is no longer tested at the final analysis and not all  $\alpha$  is spent.

⇒ **The test becomes strictly conservative!**

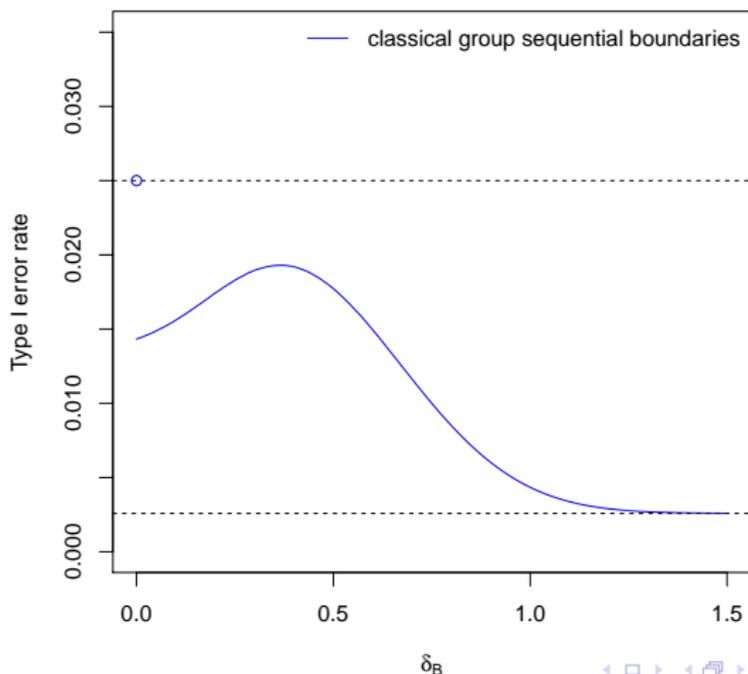
⇒ **Improved boundaries for the elementary tests possible!**

(similar as for group sequential multiple endpoint tests in Tamhane, Metha, Liu 2010).

# Why can we relax the elementary boundaries?

Example: O'Brien Fleming form of boundaries for elementary test  $H_A$ , one interim analysis after half of the patients

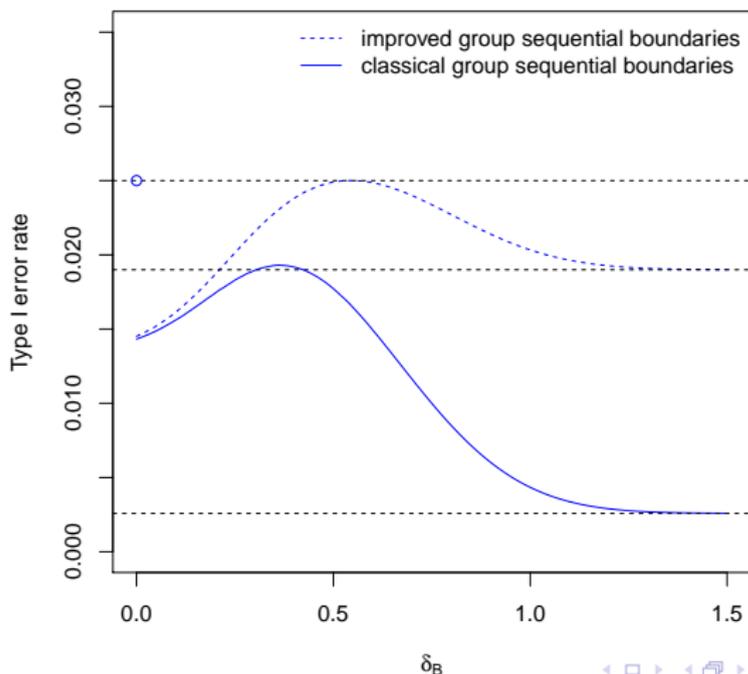
FWER for simultaneous stopping if only  $H_A$  holds ( $\delta_A=0$ )



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FWER for simultaneous stopping if only  $H_A$  holds ( $\delta_A=0$ )



## Question 2: Impact on ESS and power?

For  $\alpha = 0.025$  and  $\delta_A = \delta_B = 0.5$

Conjunctive Power = Power to reject both false hypotheses

Disjunctive Power = Power to reject at least one false hypothesis

	separate stopping rule	simultaneous stopping rule	improved simultan.
Boundaries $u_i$ for $H_1 \cap H_2$	$u_1 = 3.14, u_2 = 2.22$		
Interim boundary $v_1$	2.80	2.80	2.08
Final boundary $v_2$	1.98	1.98	1.98
Maximum $\alpha$ for test of $H_j$	0.025	0.019	0.025
Disj. power	0.97	0.97	0.97
N	324	324	324
ESS	230	205	205
Conj. power	0.89	0.69	0.76

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## Optimized multi-arm multi-stage designs

# How to optimize the designs?

Design	<b>“Separate stopping”</b>	<b>“Simultaneous stopping”</b>	<b>“Improved simult. stopping”</b>
Boundaries	group sequential	group sequential	improved group sequential
Stopping rule	<b>separate</b> stopping rule	<b>simultaneous</b> stopping rule	<b>simultaneous</b> stopping rule

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Stopping rule	separate stopping rule	simultaneous stopping rule	simultaneous stopping rule
$N_{max}$	chosen to achieve disjunctive power of 0.9		
Obj. function to optimize $u_1, u_2$	expected sample size		

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Stopping rule	separate stopping rule	simultaneous stopping rule	simultaneous stopping rule
$N_{max}$	chosen to achieve disjunctive power of 0.9		
Obj. function to optimize $u_1, u_2$	expected sample size		
Obj. function to optimize $v_1, v_2$	expected sample size	conjunctive power	

# Numerical example

Optimization for  $\delta_A = 0.5$ ,  $\delta_B = 0.5$ ,  $\alpha = 0.025$

	separate	simultaneous	improved simult.
$u_1$	2.47	2.41	2.41
$u_2$	2.38	2.43	2.43
$v_1$	2.05	2.06	2.00
$v_2$	2.38	2.37	2.06
Disj. power	0.97	0.97	0.97
N	318	324	324
ESS	225	205	205
Conj. power	0.85	0.71	0.76

# Summary

- The **optimal design** depends on the type of objective:
  - Reject **all** hypotheses
  - Reject **at least one** hypothesis
- **Simultaneous stopping compared to separate stopping** leads to
  - lower expected sample size
  - the same power to reject any hypothesis
  - lower power to reject both hypotheses

**Improved boundaries** can be used to regain some of the power to reject both null hypotheses.

- **Limitation:** If improved boundaries are used, the simultaneous stopping rule must be adhered to!
- **Extensions:**
  - more treatment arms, stopping for futility
  - optimal choice of first stage sample size/allocation ratio

## References

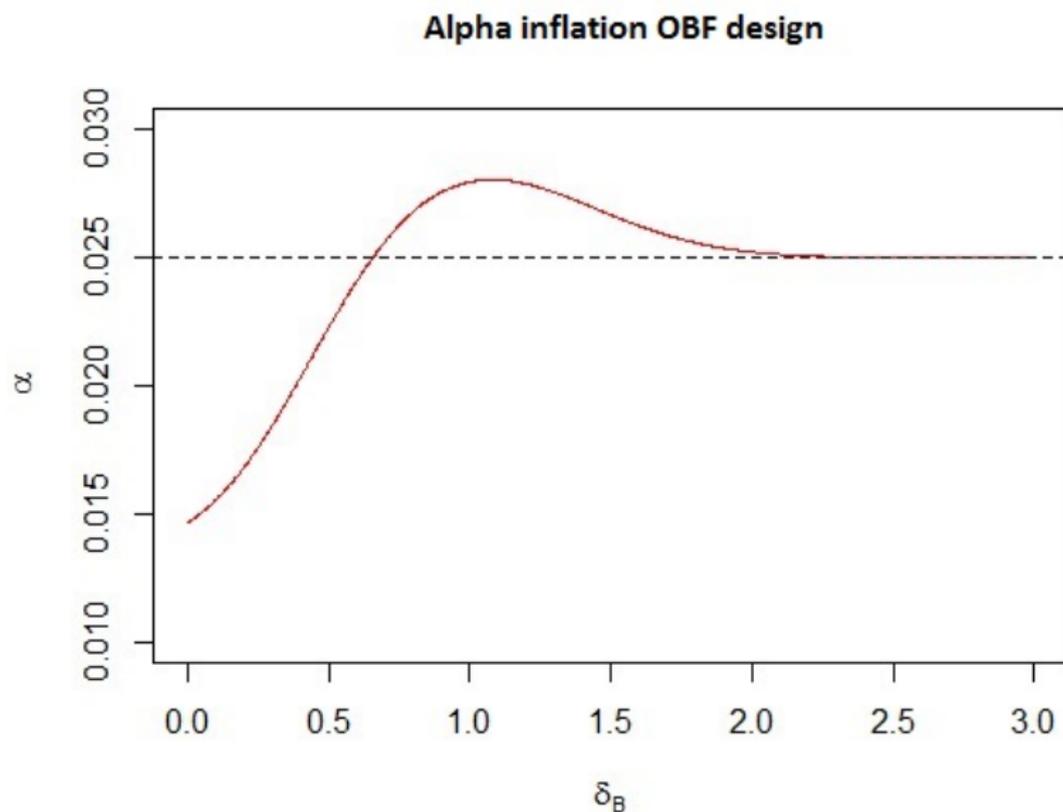
- Thall et al. (1989): one treatment continues, futility stopping, two stages, power comparisons under LFC
- Follmann et al. (1994): Pocock and OBF MAMS designs, Dunnett and Tukey generalisations, several stages
- Stallard & Todd (2003): only one treatment is taken forward, several stages, power comparisons
- Stallard & Friede (2008): stagewise prespecified number of treatments
- **Magirr, Jaki, Whitehead (2012)**: FWER of generalised Dunnett
- Koenig, Brannath, Bretz (2008): closure principle for Dunnett test, adaptive Dunnett test
- **Magirr, Stallard, Jaki (2014)**: Flexible sequential designs
- Di Scala & Glimm (2011): Time to event endpoints
- **Wason & Jaki (2012)**: Optimal MAMS designs
- **Tamhane & Xi (2013)**: multiple hypotheses and closure principle
- Maurer & Bretz (2013): Multiple testing using graphical approaches

## Unknown variance: Extension to the t test

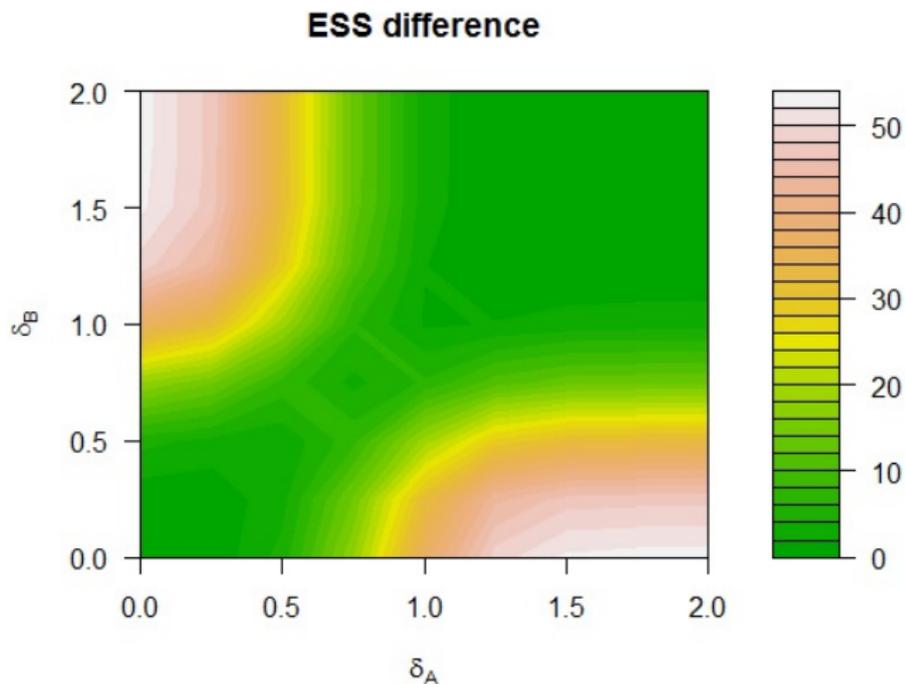
- P-value approach: z-score boundaries are converted to p-value boundaries and then applied to t-test p-values
- Simulation of t-statistics for p-value approach (optimized for  $\delta_A = \delta_B = 1$ ) for  $\sigma = 1$ .

Design	N	$\alpha$
separate	8	0.0259
	12	0.0257
	100	0.0251
improved	8	0.0261
	12	0.0258
	100	0.0250

FWER inflation when  $u_1^* = z_{1-\alpha} = 1.96$



## Difference in expected sample size: OBF design



# Difference in conjunctive power: OBF design

