

Multi-arm Group Sequential Designs with a Simultaneous Stopping Rule

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ASTERIX Project - <http://www.asterix-fp7.eu/>

Objectives of multi-arm multi-stage trials

Aim: Comparison of several treatments to a common control

Compared to separate, fixed sample two-armed trials

- less patients needed than for separate controlled clinical trials
- larger number of patients are randomised to experimental treatments
- possibility to stop early for efficacy or futility

Objective: Identify **all** treatments that are superior to control

Objective: Identify **at least one** treatment that is superior to control

Which stopping rule?

Design setup: group sequential Dunnett test

- Comparison of two treatments to a control
- Normal endpoints, variance known
- One sided tests: $H_A : \mu_A - \mu_C \leq 0$ and $H_B : \mu_B - \mu_C \leq 0$
- Control of the FamilyWise Error Rate (FWER) = 0.025
- Two stage group sequential trial: one interim analysis at $\frac{N_{max}}{2}$
- $Z_{A,i}$, $Z_{B,i}$ are the cumulative z-statistics at stage $i=1,2$

Classical group sequential Dunnett tests with “separate stopping”

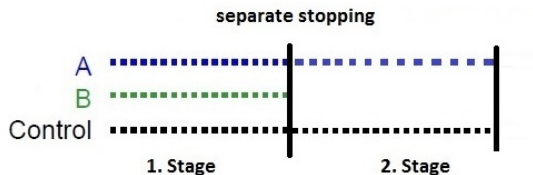
Classical group sequential Dunnett tests

Objective: Identify all treatments that are superior to control

“separate stopping rule”:

Treatment arms, for which a stopping boundary is crossed, stop.

E.g.:



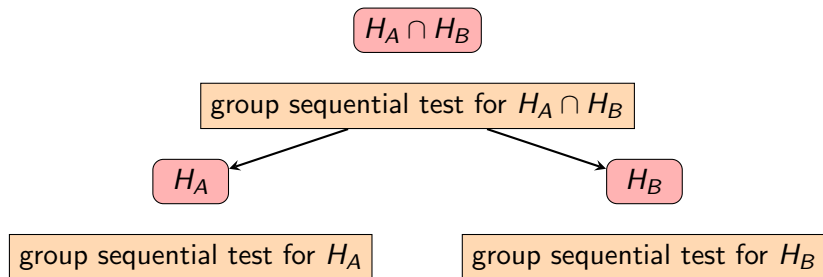
→ H_B is rejected at interim

→ A can go on and is tested again at the end

Magirr, Jaki, Whitehead (2012)

Closed group sequential tests

Local group sequential tests for $H_A \cap H_B$ and H_A, H_B are needed!!!



A hypothesis is rejected with FWER α if the intersection hypothesis and the corresponding elementary hypothesis are rejected locally at level α .

Closed group sequential tests

$$H_A \cap H_B$$

Reject if $\max(Z_{A,1}, Z_{B,1}) > u_1$ or $\max(Z_{A,2}, Z_{B,2}) > u_2$

$$H_A$$

$$H_B$$

Reject if $Z_{A,1} > v_1$ or $Z_{A,2} > v_2$

Reject if $Z_{B,1} > v_1$ or $Z_{B,2} > v_2$

u_1, u_2 ...global boundaries

v_1, v_2 ...elementary boundaries

Koenig, Brannath, Bretz and Posch (2008)

Xi, Tamhane (2015)

Maurer, Bretz (2013)

Group sequential Dunnett tests with “simultaneous stopping”

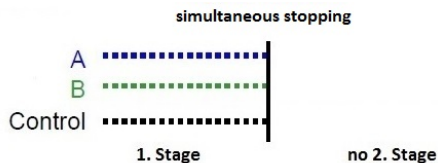
Group sequential simultaneous stopping designs

"**simultaneous stopping rule**":

If at least one rejection boundary is crossed, the whole trial stops.

Objective: Identify at least one treatment that is superior to control

If, e.g., H_B is rejected at interim then the trial is stopped:



Simultaneous versus Separate Stopping

- The **FWER** is controlled when using the boundaries of the separate stopping design.
- The **expected sample size (ESS)** is lower compared to separate stopping designs.
- The **power to reject**
 - **any** null hypothesis is the **same** as for separate stopping designs.
 - **both** null hypotheses is **lower** than for separate stopping designs.

→ **Trade-off between ESS and conjunctive power**

Construction of efficient simultaneous stopping designs

- 1 Can one **relax the interim boundaries** when stopping simultaneously?
- 2 How large is the impact on **ESS and power** when stopping simultaneously or separately?
- 3 How to **optimize** the critical boundaries for either stopping rule?

Question 1: Relaxation of interim boundaries?

For simultaneous stopping:

- The boundaries u_1, u_2 for the local test of $H_A \cap H_B$ cannot be relaxed.
- The boundaries v_1, v_2 for the local test of H_j can be relaxed.

Intuitive explanation

If, e.g., H_B is rejected at interim, but H_A not, H_A is no longer tested at the final analysis and not all α is spent.

It's possible to choose improved boundaries for the elementary tests.

(similar as for group sequential multiple endpoint tests in Tamhane, Metha, Liu 2010).

What changes when stopping simultaneously?

Example: O'Brien Fleming boundaries

$$H_A \cap H_B$$

Reject if $\max(Z_{A,1}, Z_{B,1}) > u_1$ or $\max(Z_{A,2}, Z_{B,2}) > u_2$

$$u_1 = 3.14, u_2 = 2.22$$

H_A

H_B

Reject if $Z_{A,1} > v_1$ or $Z_{A,2} > v_2$

Reject if $Z_{B,1} > v_1$ or $Z_{B,2} > v_2$

$$v_1 = 2.80, v_2 = 1.98$$

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 H_A
 H_B

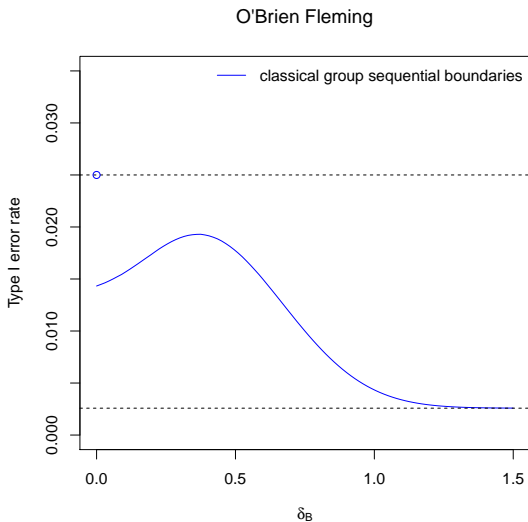
Reject if $Z_{A,1} > v_1$ or $Z_{A,2} > v_2$

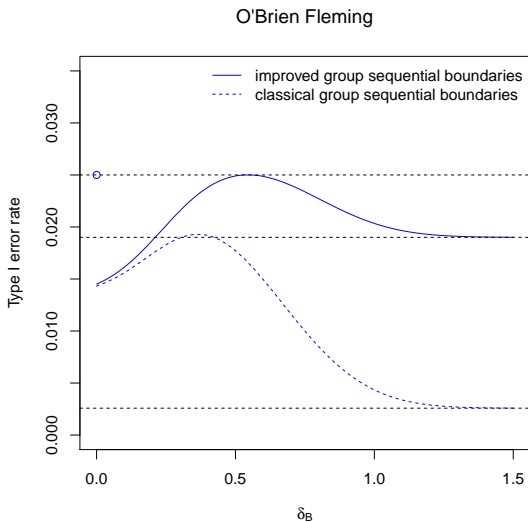
Reject if $Z_{B,1} > v_1$ or $Z_{B,2} > v_2$

$$v_1 = 2.80, v_2 = 1.98$$

$$v_1 = 2.80, v_2 = 1.98$$

For simultaneous stopping there is no second stage test if one of the null hypotheses can already be rejected at interim.

FWER for simultaneous stopping if only H_A holds ($\delta_A = 0$)

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Question 2: Impact on ESS and power?

For $\alpha = 0.025$ and $\delta_A = \delta_B = 0.5$

Conjunctive Power = Power to reject both false hypotheses

Disjunctive Power = Power to reject at least one false hypothesis

	separate stopping rule	simultaneous stopping rule	improved simultan.
Boundaries u_i for $H_1 \cap H_2$	$u_1 = 3.14, u_2 = 2.22$		
Interim boundary v_1	2.80	2.80	2.08
Final boundary v_2	1.98	1.98	1.98
Maximum α for test of H_j	0.025	0.019	0.025
Disj. power	0.97	0.97	0.97
N	324	324	324
ESS	230	205	205
Conj. power	0.89	0.69	0.76

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Optimized multi-arm multi-stage designs

Optimized designs

For $\alpha = 0.025$ and $\delta_A = \delta_B = 0.5$.

Design	“Separate stopping”	“Simultaneous stopping”	“Improved simult. stopping”
Boundaries	group sequential	group sequential	improved group sequential
Stopping rule	separate stopping rule	simultaneous stopping rule	simultaneous stopping rule

Optimized designs

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Design	“Separate stopping”	“Simultaneous stopping”	“Improved simult. stopping”
Boundaries	group sequential	group sequential	improved group sequential
Stopping rule	separate stopping rule	simultaneous stopping rule	simultaneous stopping rule
N_{max}	chosen to achieve disjunctive power of 0.9		
Obj. function to optimize u_1, u_2	expected sample size		

Optimized designs

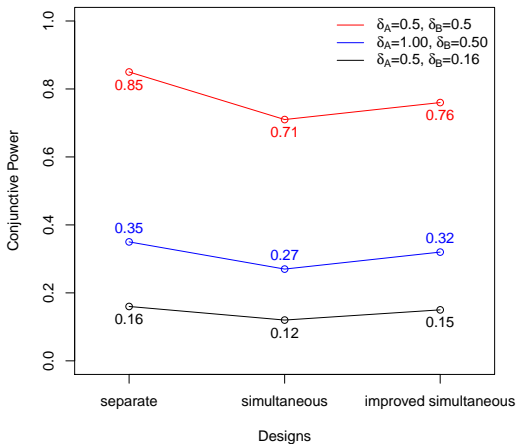
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Boundaries	group sequential	group sequential	improved group sequential
Stopping rule	separate stopping rule	simultaneous stopping rule	simultaneous stopping rule
N_{max}	chosen to achieve disjunctive power of 0.9		
Obj. function to optimize u_1, u_2	expected sample size		
Obj. function to optimize v_1, v_2	expected sample size	conjunctive power	

Optimized boundaries $\delta_A = 0.5$, $\delta_B = 0.5$

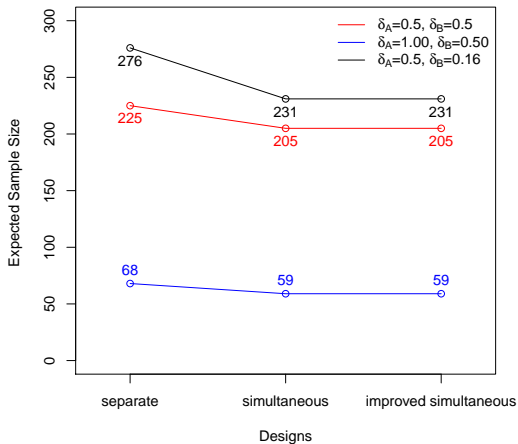
	separate	simultaneous	improved simult.
u_1	2.47	2.41	2.41
u_2	2.38	2.43	2.43
v_1	2.05	2.06	2.00
v_2	2.38	2.37	2.06
conj. power	0.85	0.71	0.76
ESS	225	205	205
N_{max}	318	324	324

Power to reject both null hypotheses



Power to reject at least one false hypothesis = 90% for all designs.

Optimal expected sample size (ESS)



ESS reduction between 8% and 16%.

Unknown variance: Extension to the t test

- P-value approach: z-score boundaries are converted to p-value boundaries and then applied to t-test p-values
- Simulation of t-statistics for p-value approach (optimized for $\delta_A = \delta_B = 1$) for $\sigma = 1$.

Design	N	α
separate	8	0.0259
	12	0.0257
	100	0.0251
improved	8	0.0261
	12	0.0258
	100	0.0250

Summary

- The **optimal design** depends on the type of objective:
 - Reject **all** hypotheses
 - Reject **at least one** hypothesis
- **Simultaneous stopping compared to separate stopping** leads to
 - lower expected sample size
 - the same power to reject any hypothesis
 - lower power to reject both hypotheses

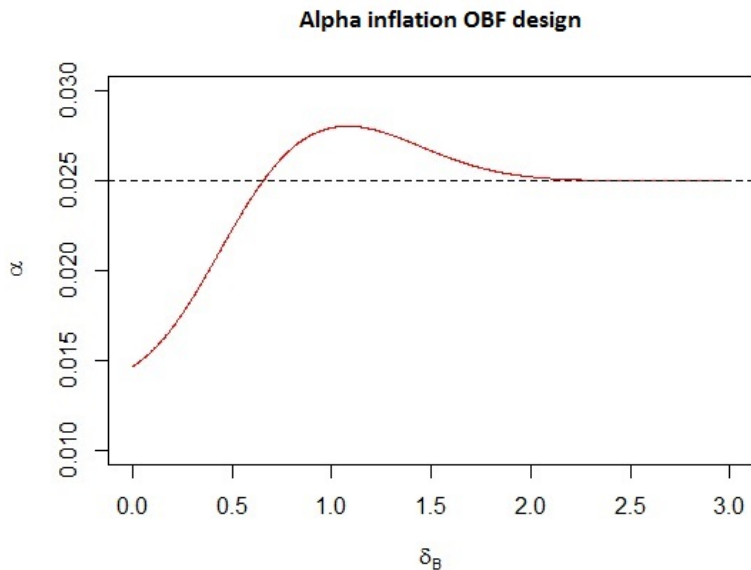
Improved boundaries can be used to regain some of the power to reject both null hypotheses.

- **Limitation:** If improved boundaries are used, the simultaneous stopping rule must be adhered to!
- **Extensions:**
 - more treatment arms, stopping for futility
 - optimal choice of first stage sample size/allocation ratio

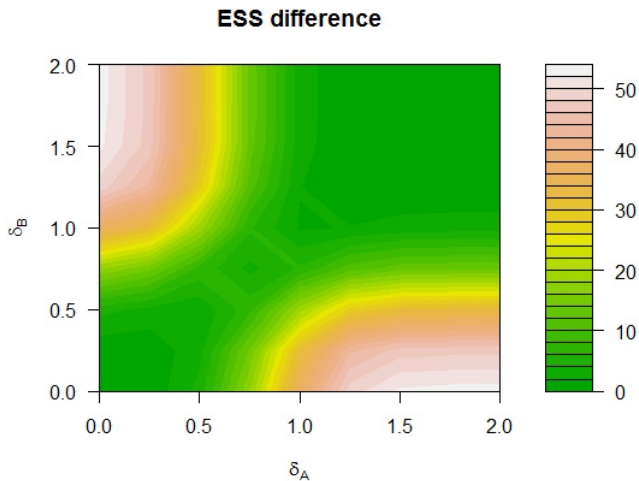
References

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- Stallard & Todd (2003): only one treatment is taken forward, several stages, power comparisons
- Stallard & Friede (2008): stagewise prespecified number of treatments
- **Magirr, Jaki, Whitehead (2012)**: FWER of generalised Dunnett
- Koenig, Brannath, Bretz (2008): closure principle for Dunnett test, adaptive Dunnett test
- **Magirr, Stallard, Jaki (2014)**: Flexible sequential designs
- Di Scala & Glimm (2011): Time to event endpoints
- **Wason & Jaki (2012)**: Optimal MAMS designs
- **Tamhane & Xi (2013)**: multiple hypotheses and closure principle
- Maurer & Bretz (2013): Multiple testing using graphical approaches

FWER inflation when $u_1^* = z_{1-\alpha} = 1.96$



Difference in expected sample size: OBF design



Difference in conjunctive power: OBF design

